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ACOUSTICS OF DUCTS WITH PLANE PERMEABLE WALLS

N. M. Khlestkina[1] **and V. Sh. Shagapov**[2]

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Theoretical foundations of pressure-waves propagation in ducts are given in [1, 2] and developed in [3, 4] with allowance for the properties of a real liquid. The most complete bibliography and a historical outline of this problem can be found in [5–7].

The goal of the present paper is to investigate the evolution of acoustic waves in liquid- (or gas)-filled ducts with porous and permeable walls. We also analyze the influence of filtration processes through permeable walls, inertial phenomena, and dissipative effects, which are related to viscous friction and heat exchange, on the peculiarities of wave propagation and damping in ducts.

1. Basic Equations. Let us consider the problem of small-perturbation propagation in ducts with plane-parallel walls under the following basic assumptions: the duct and the incompressible skeleton of the surrounding porous space are filled with the same liquid (or gas). The liquid is barotropic and its viscosity manifests itself only upon filtration. In addition, it is assumed that the duct width is infinite (i.e., the duct height is much smaller than the duct width), and the wavelength is greater than the duct height. We introduce the following coordinate system: the Oz axis is directed along the duct axis and the Ox axis is perpendicular to the upper (or lower) duct wall.

Under these assumptions, the linearized system of equations that governs the propagation of perturbations in ducts with porous and permeable walls has the form

$$\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial w}{\partial z} = -\frac{\rho_0 u}{\alpha_0}, \qquad |x| < \alpha_0; \tag{1.1}$$

$$\rho_0 \frac{\partial w}{\partial t} + \frac{\partial P}{\partial z} = 0, \qquad |x| < \alpha_0; \tag{1.2}$$

$$P = C^2 \rho; \tag{1.3}$$

$$m \frac{\partial \rho^{(1)}}{\partial t} = -\rho_0 \frac{\partial u^{(1)}}{\partial x}, \qquad |x| > \alpha_0; \tag{1.4}$$

$$u^{(1)} = -\frac{k_c}{\mu} \frac{\partial P^{(1)}}{\partial x}, \qquad |x| > \alpha_0; \tag{1.5}$$

$$u^{(1)} = u, \qquad P^{(1)} = P, \qquad |x| = \alpha_0. \tag{1.6}$$

Here P and ρ are the pressure and density perturbations, respectively; w is the velocity of the liquid at the cross section with the z coordinate at instant t; u is the filtration velocity through the duct walls; C is the sound velocity of the liquid; the subscript zero denotes an unperturbed state; $P^{(1)}$, $\rho^{(1)}$, and $u^{(1)}$ are the distributions of pressure, density, and filtration velocity perturbations in the porous space around the duct; μ is the dynamic viscosity of the liquid; k_c and m are the permeability and porosity of the space surrounding the duct; α_0 is the half-height of the duct.

From Eqs. (1.1)–(1.5) we have

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[1]Institute of Mechanics, Ural Scientific Center, Russian Academy of Sciences, Ufa 450000. [2]Institute for Mechanics of Multiphase Systems, Siberian Division, Russian Academy of Sciences, Tyumen' 625000. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 37, No. 5, pp. 82–92, September–October, 1996. Original article submitted July 11, 1995.

$$\frac{1}{C^2}\frac{\partial^2 P}{\partial t^2} - \frac{\partial^2 P}{\partial z^2} = \frac{\rho_0 \partial u}{\alpha_0 \partial t}, \qquad |x| < \alpha_0; \tag{1.7}$$

$$\frac{\partial P^{(1)}}{\partial t} = æ \frac{\partial^2 P^{(1)}}{\partial x^2}, \qquad æ = \frac{k_c \rho_0 C^2}{\mu m}, \qquad |x| > \alpha_0; \tag{1.8}$$

$$P = P^{(1)}, \qquad u = -\frac{k_c}{\mu}\frac{\partial P^{(1)}}{\partial x}, \qquad |x| > \alpha_0,$$

where æ is the pressure conductivity.

A wave equation that governs the dynamics of small perturbations in ducts with infinitely thick walls can be derived for the problem in question from Eq. (1.7) and the pressure conduction equation subject to boundary conditions (1.6).

The initial and boundary conditions

$$\begin{aligned} &P^{(1)} = 0, \qquad t = 0, \qquad |x| > \alpha_0, \\ &P^{(1)} = P, \qquad t > 0, \qquad |x| = \alpha_0 \end{aligned} \tag{1.9}$$

should be added to Eq. (1.8) to determine uniquely the process under study.

In accordance with Duhamel's principle, the solution of Eq. (1.8) with specified conditions (1.9) has the form [8]

$$P^{(1)} = \int_0^t \frac{\partial U}{\partial t}(x - \alpha_0, t - \tau) P(z, \tau)\, d\tau. \tag{1.10}$$

where $U(x, t)$ is a solution of a similar boundary-value problem for $U(0, t) = 1$.

Since

$$U(x - \alpha_0, 0) = 0, \tag{1.11}$$

it follows from (1.5) with allowance for (1.10) and (1.11) at $|x| = \alpha_0$ that

$$u = \frac{k_c}{\mu\sqrt{\pi æ}} \frac{\partial}{\partial t} \int_0^t \frac{P(z, \tau)}{\sqrt{t - \tau}}\, d\tau. \tag{1.12}$$

Substituting (1.12) into (1.7), we obtain the wave equation

$$\frac{1}{C^2}\frac{\partial}{\partial t^2}\left(P + \frac{m\sqrt{æ}}{\sqrt{\pi}\alpha_0} \int_0^t \frac{P(z, \tau)}{\sqrt{t - \tau}}\, d\tau\right) - \frac{\partial^2 P}{\partial z^2} = 0, \tag{1.13}$$

which governs the evolution of small perturbations in semi-infinite ducts with porous and permeable walls. The assumption that the medium is initially (at $t = 0$) at rest ($P^{(1)} = P = 0$) is essential for deriving the equation.

In the most general form corresponding to the condition $P^{(1)} = P = 0$ for $t = -\infty$, Eq. (1.13) can be written as

$$\frac{1}{C^2}\frac{\partial}{\partial t^2}\left(P + \frac{m\sqrt{æ}}{\sqrt{\pi}\alpha_0} \int_{-\infty}^t \frac{P(z, \tau)}{\sqrt{t - \tau}}\, d\tau\right) - \frac{\partial^2 P}{\partial z^2} = 0. \tag{1.14}$$

The wave equation (1.13) [or (1.14)] obtained above corresponds to a duct with porous and permeable walls of infinite thickness. If the wall thickness is finite, two limiting cases are possible: 1) the outer surface of the duct wall ($|x| = \alpha_*$, $\alpha_* > \alpha_0$) is contiguous with impermeable space, and 2) the outer boundary of the

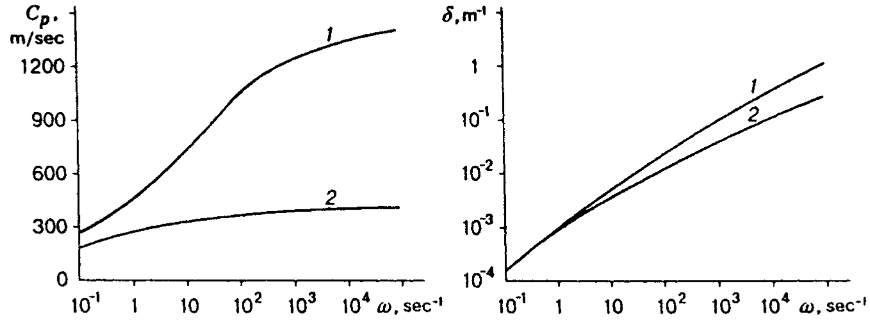

Fig. 1

duct wall is a free surface. The corresponding boundary conditions have the form

$$u^{(1)} = -\frac{k_c}{\mu}\frac{\partial P^{(1)}}{\partial x} = 0, \qquad |x| = \alpha_*$$

or

$$P^{(1)} = 0, \qquad |x| = \alpha_*,$$

where α_* is the position of the outer boundary of the duct wall.

2. Dispersion Analysis. We seek a solution of the problem in the form of a traveling damped wave. We assume that the wave propagates along the z axis in the positive direction:

$$P = A_p \exp[i(Kz - \omega t)], \quad u = A_u \exp[i(Kz-\omega t)], \quad P^{(1)} = A_p^{(1)}(x)\exp[i(Kz-\omega t)],$$
$$u^{(1)} = A_u^{(1)}(x)\exp[i(Kz-\omega t)], \quad K = k + i\delta, \quad C_p = \omega/k.$$

Here K is the complex wave number; δ is the linear damping increment; C_p is the phase velocity; i is an imaginary unit; and ω is the angular frequency of perturbations.

From the condition for the existence of a solution of this type for ducts with infinitely thick walls, we obtain the dispersion expression

$$K^2 = \omega^2(1 + m/y)C^{-2}, \qquad y = \sqrt{-i\omega\alpha_0^2/æ}, \tag{2.1}$$

where the parameter $|y| = \sqrt{\omega\alpha_0^2/æ} = \alpha_0/\alpha_\omega$ $(\alpha_\omega = \sqrt{æ/\omega})$ has the physical meaning [8] of the ratio of the duct half-height α_0 to the depth of penetration α_ω of filtration waves with frequency ω into the porous space.

An analysis of the dispersion expression (2.1) gives the asymptotic relations

$$C_p \simeq \left(\frac{\alpha_0 C^2\nu}{k_c m}\omega\right)^{1/4}, \qquad \delta \simeq \frac{\pi}{8}\left(\frac{m k_c \omega^3}{C^2\alpha_0^2\nu}\right)^{1/4}, \qquad \nu = \frac{\mu}{\rho_0} \tag{2.2}$$

(ν is the kinematic viscosity of the medium) for low frequencies for which either the condition $m/|y| \gg 1$ or $\sqrt{\omega} \ll m\sqrt{\omega_æ} = \sqrt{\omega_c}$ is satisfied ($\omega_æ = æ/\alpha_0^2$ is the characteristic frequency such that the penetration depth of filtration waves is on the order of the duct half-height [8]).

For high frequencies that satisfy either the condition $m/|y| \ll 1$ or $\sqrt{\omega} \gg \sqrt{\omega_c}$, the asymptotic relations have the form

$$C_p \simeq C, \qquad \delta = \frac{1}{2\sqrt{2}\alpha_0}\sqrt{\frac{k_c m}{\nu}\omega}. \tag{2.3}$$

An analysis of expressions (2.2) and (2.3) shows that the propagation velocity of harmonic waves in ducts with plane walls of infinite thickness varies from zero ($C_p \ll C$) in the low-frequency range ($\sqrt{\omega} \ll \sqrt{\omega_c}$) to a value close to the sound velocity of the medium ($C_p \simeq C$) in the high-frequency range ($\sqrt{\omega} \gg \sqrt{\omega_c}$). The quantity $\sqrt[4]{C^2\nu}$ is a physical parameter of the medium that governs the evolution of low-frequency

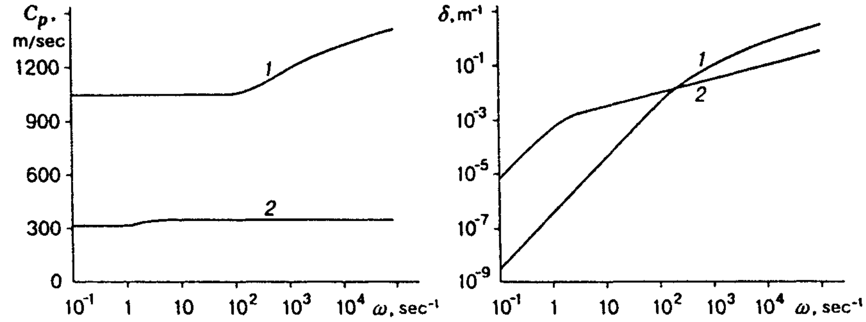

Fig. 2

perturbations in such ducts; and the damping intensity increases with increasing compressibility of the medium (which is determined by the sound velocity C). In the high-frequency range, when the phase velocity of the wave is close to the sound velocity in the medium, perturbation damping is determined by the kinematic viscosity of the medium ν, wave damping being stronger in less viscous media. The damping coefficient of both low- and high-frequency perturbations is inversely related to the duct height (the thinner the duct, the more intense the damping process) and is directly related to the porosity and permeability of the duct walls. This relation is stronger in the high-frequency range.

Figure 1 presents the propagation velocity and the damping coefficient calculated by the dispersion equation (2.1) as functions of frequency in ducts ($\alpha_0 = 5 \cdot 10^{-3}$ m, $k_c = 10^{-12}$ m^2, and $m = 0.2$) filled with water (curve 1, $\nu = 1.06 \cdot 10^{-6}$ m^2/sec and $C = 1.425 \cdot 10^3$ m/sec) and air (curve 2, $\nu = 1.5 \cdot 10^{-5}$ m^2/sec and $C = 3.41 \cdot 10^2$ m/sec). It is seen that damping of low-frequency perturbations is approximately the same in water and air. In the high-frequency range, the damping process is more active in water. For the case under consideration, the ratio of the damping coefficients of perturbations in water δ_w and in air δ_a is $\delta_w/\delta_a \simeq 4$ for the frequencies at which the wave propagation velocity is close to the sound velocity in the medium ($C_p \simeq C$).

For ducts with finitely thick walls, when the outer surface of the porous wall is contiguous with impermeable space, the dispersion expression has the form

$$K^2 = \omega^2\Big(1 + \frac{m}{y}\tanh\big(y(A_* - 1)\big)\Big)C^{-2}, \qquad A_* = \frac{\alpha_*}{\alpha_0}. \tag{2.4}$$

In a low-frequency range for which the condition

$$|y|(A_* - 1) \ll 1 \quad \text{or} \quad \sqrt{\omega} \ll \sqrt{\omega_{æ}^{(e)}}, \quad \omega_{æ}^{(e)} = æ/(\alpha_* - \alpha_0)^2 \tag{2.5}$$

is satisfied ($\omega_{æ}^{(e)}$ is the characteristic frequency at which the depth of penetration of filtration waves into the porous medium is on the order of the porous-wall thickness), for the phase velocity and the damping coefficient we obtain

$$C_p \simeq C^{(e)} = \frac{C}{\sqrt{1 + m(A_* - 1)}}, \qquad \delta = \frac{m(A_* - 1)}{6C^{(e)}(1 + m(A_* - 1))}\,\frac{\omega^2\alpha_0^2}{æ}. \tag{2.6}$$

It is seen from expressions (2.6) that low-frequency perturbations ($\sqrt{\omega} \ll \sqrt{\omega_{æ}^{(e)}}$) (for which the penetration depth of filtration waves is much greater than the thickness of the porous wall of the duct $\alpha_\omega \gg \alpha_* - \alpha_0$) in ducts with finitely thick walls surrounded by poorly permeable space propagate at a characteristic velocity $C^{(e)}$. This velocity is determined by the sound velocity in the medium and by the geometrical parameters of the duct (the height and thickness of the walls) and does not depend on the perturbation frequency. In this case, the wave-propagation velocity is the higher, the greater the height of the duct and the thinner its walls. The process of perturbation damping is determined by the quantity ν/C^3, i.e., by the kinematic viscosity and compressibility of the medium, the dependence on compressibility being stronger. The intensity of perturbation damping in the low-frequency range increases with an increase in the porosity of the duct

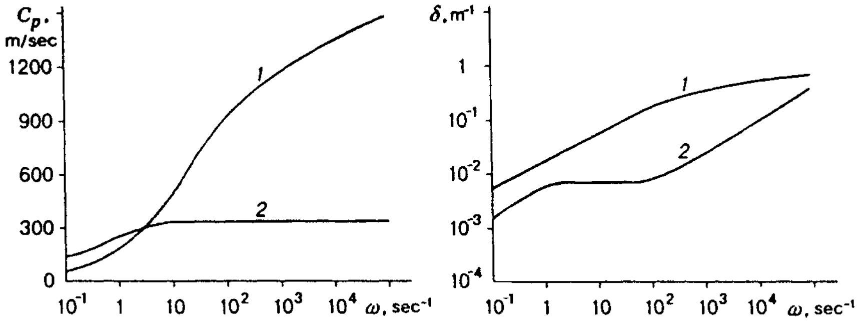

Fig. 3

walls and decreases with an increase in their permeability.

Figure 2 presents the propagation velocity and the damping coefficient obtained from expression (2.4) as dispersion functions of frequency in water- and air-filled (curves 1 and 2) ducts ($\alpha_0 = 10^{-2}$ m) surrounded by poorly permeable porous space. It is seen that the damping of low-frequency perturbations in air is faster than in water. The ratio of the wave-damping coefficients in water and air in this case is $\delta_w/\delta_a \simeq 3$ for $m = 0.2$ and $k_c = 10^{-12}$ m^2.

When the thickness of the absorption zone is comparable with the thickness of the duct wall ($\alpha_\omega \simeq \alpha_* - \alpha_0$) and the outer boundary of the duct wall is a free surface, we obtain the dispersion expression

$$K^2 = \omega^2\Big(1 + \frac{m}{y \tanh(y(A_* - 1))}\Big)C^{-2}. \tag{2.7}$$

From this equation for low frequencies that satisfy condition (2.5) we have

$$K^2 = \omega^2\Big(1 + \frac{m}{(A_* - 1)}\frac{1}{y^2}\Big)C^{-2}.$$

For frequencies satisfying, apart from (2.5), the condition $|y|^2 \ll m/(A_* - 1)$ or $\omega \ll \omega_æ m/(A_* - 1) = \omega_æ^*$ ($\omega_æ^*$ is the characteristic frequency, at which $C_p \simeq C$) the propagation velocity and the damping coefficient are written as

$$C_p = \sqrt{\frac{2(\alpha_* - \alpha_0)\alpha_0\omega\nu}{k_c}}, \qquad \delta = \sqrt{\frac{k_c\omega}{2(\alpha_* - \alpha_0)\alpha_0\nu}}.$$

For frequencies satisfying, apart from (2.5), the condition $|y|^2 \gg m/(A_* - 1)$ or $\omega \gg \omega_æ^*$, we obtain the asymptotic relations $C_p = C$, $\delta = Ck_c/[2\alpha_0(\alpha_* - \alpha_0)\nu]$

Thus, the results of the analysis show that the propagation velocity of low-frequency perturbations in ducts with finitely thick walls in a highly permeable medium varies from zero in the frequency range $\omega \ll \omega_æ^*$ to a value that is close to the sound velocity in the medium in the frequency range $\omega \gg \omega_æ^*$. The quantity C/ν is a key physical parameter that governs the damping process. The damping intensity is inversely related to the kinematic viscosity and compressibility of the medium and to the geometrical parameters of the duct (the height and thickness of the walls).

Figure 3 shows the propagation velocity and the damping coefficient of the wave as functions of its frequency in water- and air-filled (curves 1 and 2) ducts ($\alpha_0 = 10^{-2}$ m, $A_* = 1.5$, $k_c = 10^{-12}$ m^2, and $m = 0.2$). It is seen from the graphic representation of the dispersion expression (2.7) that damping of low-frequency perturbations in water ($C_1/\nu_1 \simeq 1.4 \cdot 10^9$ m^{-1}) is stronger than in air ($C_2/\nu_2 \simeq 2.3 \cdot 10^7$ m^{-1}).

For low-frequency perturbations, a frequency range ($\omega \gg \omega_æ^*, \sqrt{\omega} \ll \sqrt{\omega_æ^{(e)}}$) exists in which the wave damping coefficient does not depend on its frequency. The left boundary of this range is determined by the characteristic frequency $\omega_æ^*$ at which the wave propagation velocity is close to the sound velocity in the medium ($C_p \simeq C$), and the right boundary is determined by the characteristic frequency $\omega_æ^{(e)}$ at which the penetration depth of filtration perturbations is on the order of the thickness of the duct porous wall. This range can be defined as $\omega_{æ}^{(e)}/\omega_{æ}^{*} = \alpha_0/[m(\alpha_* - \alpha_0)]$. The existence of frequencies for which the damping coefficient of the perturbation does not depend on its frequency is observed in thin-walled ducts and is not observed with an increase in the wall thickness. The frequency range can be large, as, in particular, in the case shown in Fig. 3, where $\omega_{æ}^{(e)}/\omega_{æ}^{*} \simeq 100$.

The high-frequency asymptotics obtained for ducts with porous walls of finite thickness in both poorly permeable space and highly permeable space coincide with the dispersion expression that characterizes the dynamics of high-frequency perturbations in ducts with infinitely thick walls. It is seen from Figs. 2 and 3 that damping of high-frequency perturbations propagating at a velocity close to the sound velocity in a medium is determined by the kinematic viscosity of the medium (in a lower viscous medium (water), wave damping is more intense than in air). In the high-frequency range, wave damping is the stronger, the higher the porosity and permeability of the duct walls.

3. Evolution of a Stepwise Wave. Let us consider the propagation of a weak shock wave within the framework of the wave equation (1.14). Let the pressure at the boundary of a semi-infinite duct increase by $P_e = \text{const}$ at $t = 0$. We investigate the evolution of this perturbation.

Assuming that the medium in the duct is at rest in the initial state at a pressure P_0 ($P = 0$), the mathematical problem of the pressure distribution $P(z,t)$ in a semi-infinite duct with porous and permeable walls can be formulated by adding to (1.14) additional initial and boundary conditions of the form

$$\begin{aligned} P = 0, \qquad \partial P/\partial t = 0, \qquad x \geqslant 0, \quad t = 0, \\ P = P_e, \qquad\qquad x = 0, \quad t > 0. \end{aligned} \tag{3.1}$$

Solving the problem by the Laplace transform method with respect to time (considering only the right half-plane $\operatorname{Re}\lambda > 0$), we go over from system (1.13) and (3.1) to an operator representation of the problem:

$$\frac{\partial P}{\partial z^2} = \hat{k}^2 \hat{P}, \quad \hat{P}(z,\lambda) = \int\limits_0^\infty P(z,t)\exp(-\lambda t)\,dt, \tag{3.2}$$

$$\hat{k}^2 = (\lambda^2 C^{-2}(1 + \beta/\sqrt{\lambda})), \quad \operatorname{Re}\hat{k} \geqslant 0, \quad -\pi/2 \leqslant \arg\lambda \leqslant \pi/2, \quad \beta = \frac{m}{\alpha_0}\sqrt{\frac{æ}{\pi}};$$

$$\hat{P}(0,\lambda) = P_e/\lambda. \tag{3.3}$$

The solution of Eq. (3.2) satisfying condition (3.3) is written as $\hat{P}(z,\lambda) = P_e\exp(-\hat{k}z)/\lambda$. Hence, by using the Mellin integral, the solution of the equation of the evolution of small perturbations in semi-infinite ducts (1.14) for a stepwise wave has the form

$$P(z,t) = \frac{P_e}{2\pi i}\int\limits_{s-i\infty}^{s+i\infty} \exp[\lambda(t - zC^{-1}(1 + \beta/\sqrt{\lambda}))^{1/2}]\lambda^{-1}\,d\lambda. \tag{3.4}$$

It can be shown that the pressure perturbations are equal to zero $[P(z,t) = 0]$ for the space and time coordinates satisfying the condition $z > Ct$. Physically, this means that the wave leading edge propagates at a velocity that does not exceed the sound velocity in the medium.

In Eq. (3.4), we perform integration along the imaginary axis ($s \to 0$), bypassing the coordinate origin ($\lambda = 0$) which is a first-order pole for the integrand. In this case ($\lambda = i\sigma$), formula (3.4) takes the form

$$\begin{gathered} P(z,t) = \frac{P_e}{2} + \frac{P_e}{\pi}\int\limits_0^\infty \exp[-\sigma z C^{-1}\sqrt{\psi}\sin(\varphi/2)]\sin[\sigma(t - zC^{-1}\sqrt{\psi}\cos(\varphi/2))]\sigma^{-1}\,d\sigma, \\ y = z/Ct, \qquad \varphi = \arctan\Big(\frac{\beta}{\beta + \sqrt{2\sigma}}\Big), \qquad \psi = \sqrt{\Big(1 + \frac{\beta}{\sqrt{2\sigma}}\Big)^2 + \Big(\frac{\beta}{\sqrt{2\sigma}}\Big)^2}. \end{gathered} \tag{3.5}$$

Analysis of (3.3) shows that the compressibility of the medium (determined by the sound velocity)

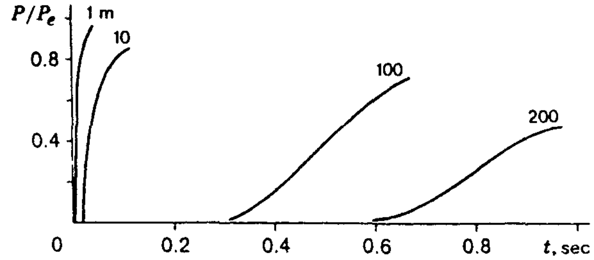

Fig. 4

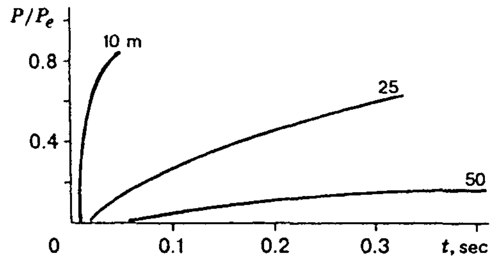

Fig. 5

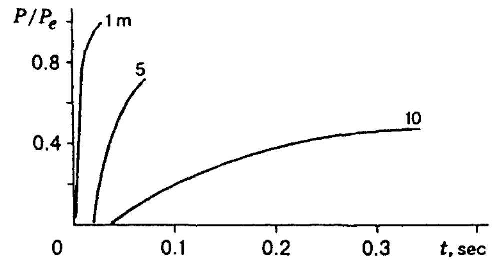

Fig. 6

is a governing physical parameter which influences the evolution of long-wave perturbations in semi-infinite channels with porous and permeable walls. The height of the duct has a substantial influence on the damping process (smearing of the wave leading edge): the smaller the duct height, the more intense the damping process.

Figures 4 and 5 show the evolution diagrams of the shock wave in air- and water-filled ducts ($\alpha_0 = 5 \cdot 10^{-2}$ m, $m = 0.2$, and $k_c = 10^{-12}$ m^2), which were obtained by numerical realization of the solution of Eq. (3.5). Figure 6 illustrates the propagation of this perturbation in an air-filled duct whose height is an order of magnitude smaller ($\alpha_0 = 5 \cdot 10^{-3}$ m). The curve numbers denote the distance from the point of signal initiation.

It is seen from Figs. 4 and 6 that, in an air-filled duct 10 cm high, the amplitude of the leading edge of the shock wave decreases approximately by a factor of 2 at a distance of 200 m from the point of signal initiation. In a duct 10 mm high, the wave amplitude is approximately 40% of the initial amplitude even at a distance of 10 m from the point of signal initiation.

A comparison of Figs. 4 and 5 shows that the amplitude of the wave leading edge in water decreases by approximately a factor of 2 at a distance of 25 m from the point of signal initiation, and approximately the same wave-amplitude reduction is observed at a distance of 200 m in the air-filled duct.

4. Influence of Viscosity, Heat Losses, and Inertial Effects. To analyze the influence of viscosity on the perturbation dynamics in ducts, we assume that the influence of viscosity manifests itself only in a thin boundary layer near the duct wall, whose thickness is much smaller than the duct height, and write the momentum equation taking into account the viscous friction:

$$\rho_0 \frac{\partial w}{\partial t} + \frac{\partial P}{\partial z} = -\frac{\tau}{\alpha_0}. \tag{4.1}$$

Here τ is the tangential viscous stress on the inner surface of the duct wall, which can be determined for this model from the relation $\tau = -\mu(\partial w'/\partial x)\alpha_0$, where $w'(x, z, t)$ is the true velocity distribution for the duct cross section with the z coordinate at instant t.

Using the Navier–Stokes equation for the true velocity distribution and performing operations similar to those in deriving expression (1.12), we obtain, instead of (4.1),

$$\rho_0 \frac{\partial}{\partial t}\left(w + \frac{\sqrt{\nu}}{\sqrt{\pi}\alpha_0}\int_{-\infty}^{t} \frac{w}{\sqrt{t-\tau}}\,d\tau\right) + \frac{\partial P}{\partial z} = 0. \tag{4.2}$$

With allowance for (1.12) the continuity equation (1.4) can be reduced to the form

$$\frac{1}{C^2}\frac{\partial}{\partial t}\left(P + \frac{\sqrt{\nu}m}{\sqrt{\pi}\alpha_0}\int_{-\infty}^{t} \frac{P}{\sqrt{t-\tau}}\,d\tau\right) + \rho_0 \frac{\partial w}{\partial z} = 0. \tag{4.3}$$

The dispersion equation for system (4.2) and (4.3) is

$$K^2 = \omega^2(1 + m/y)(1 + 1/y_{(\nu)})C^{-2}, \qquad y_{(\nu)} = \sqrt{-i\omega\alpha_0^2/\nu}\,. \tag{4.4}$$

To estimate the influence of the effects due to temperature nonequilibrium on the perturbation dynamics, we assume that the heat-exchange intensity is limited by the thermal resistance of the gas medium (because the thermal conductivity is usually considerably smaller for gas than for solids), and write the equation of heat gain for the medium in the duct:

$$\rho_0 c_g \frac{\partial T}{\partial t} = \frac{\partial P}{\partial t} - \frac{q}{\alpha_0}.$$

Here T is the temperature, c_g is the specific heat of the gas at a constant pressure; and q is the heat-exchange intensity per unit area of the duct wall. Assuming that the temperature drop occurs in a thin boundary layer whose thickness is much smaller than the duct height, far away from the duct wall where the gas behavior is nearly adiabatic, we obtain

$$\rho_0 c_g \frac{\partial T^{\mathrm{ad}}}{\partial t} = \frac{\partial P}{\partial t} \qquad \left(\frac{T^{\mathrm{ad}}}{T_0} = \frac{\gamma - 1}{\gamma}\frac{P}{P_0} \quad \text{or} \quad T^{\mathrm{ad}} = \frac{P}{\rho_0 c_g}\right), \tag{4.5}$$

where γ is the adiabatic exponent of the gas. We write the equation

$$\rho_0 c_g \frac{\partial T'}{\partial t} = \lambda \frac{\partial^2 T'}{\partial x^2} + \frac{\partial P}{\partial t}, \qquad |x| < \alpha_0. \tag{4.6}$$

for the temperature distribution $T'(z, x, t)$ near the duct wall.

It follows from Eqs. (4.5) and (4.6) that $\partial \Delta T/\partial t = æ_T \partial^2 \Delta T/\partial x^2$, $\Delta T = T' - T^{\mathrm{ad}}$, $æ_T = \lambda/(\rho_0 c_g)$. After manipulations similar to those performed in deriving expression (1.13), from the continuity equation (1.1) with allowance for (1.2) we obtain the wave equation

$$\frac{1}{C^2}\frac{\partial^2}{\partial t^2}\left(P + \frac{m\sqrt{æ} + (\gamma - 1)\sqrt{æ_T}}{\sqrt{\pi}\alpha_0}\int_0^t \frac{P(z,\tau)}{\sqrt{t-\tau}}\,d\tau\right) - \frac{\partial^2 P}{\partial x^2} = 0$$

and the corresponding dispersion expression

$$K^2 = \omega^2\left(1 + \frac{m}{y} + \frac{\gamma - 1}{y_{(T)}}\right)C^{-2}, \qquad y_{(T)} = \sqrt{-i\,\frac{\omega\alpha_0^2}{æ_T}}\,. \tag{4.7}$$

It follows from expressions (4.4) and (4.7) that the effect of energy dissipation caused by viscosity and heat exchange on the evolution of waves can be compared with the effect of filtration using the relations

$$Skh^{(\nu)} = \frac{1/y_{(\nu)}}{m/y} = \frac{1}{m}\sqrt{\frac{\nu}{æ}}, \qquad Skh^{(T)} = \frac{\gamma - 1}{m}\sqrt{\frac{æ_T}{æ}}.$$

It is evident that in most cases of practical interest, $Skh^{(\nu)}$, $Skh^{(T)} \ll 1$ and, consequently, the effect of energy dissipation caused by friction and heat exchange between the medium in the duct and its walls can be ignored. We used Darcy's law as a basis to construct a theoretical model of the filtration process through permeable walls. However, inertial effects upon liquid filtration in porous space can have an influence on the evolution of short-time perturbations in ducts with permeable walls. To analyze this influence, we use the momentum equation in the most general form [9]:

$$\rho_0 \frac{\partial u^{(1)}}{\partial t} = -m \frac{\partial P^{(1)}}{\partial x} - \frac{m\mu}{k_c} u^{(1)}.$$

After appropriate transformations we obtain the dispersion equation

$$K^2 = \omega^2 \Big(1 + \frac{m}{y\sqrt{1 - i\omega/\omega_i}}\Big) C^{-2}. \tag{4.8}$$

A comparison of (4.8) and expression (2.1) shows that inertial effects can have a substantial influence on the evolution of perturbations whose frequencies satisfy the condition $\omega \gg \omega_i = m\nu/k_c$. In particular, for a porous water-filled medium at $m = 0.1$, $k_c = 10^{-12}$ m^2, and $\nu = 10^{-6}$ m^2/sec the characteristic frequency is $\omega_i \simeq 10^5$ sec^{-1}.

It follows from the basic assumption adopted above ($\lambda = 2\pi C/\omega \geqslant 2\alpha_0$) that the theoretical model is applicable to perturbations whose frequencies are determined by the condition $\omega \ll \omega_* = \pi C/\alpha_0$. In particular, for a porous water-filled medium at $\alpha_0 = 10^{-2}$ m the frequency that limits the applicability range of the theoretical model is $\omega_* = 3 \cdot 10^5$ sec^{-1}.

Consequently, the frequency range in which inertial effects associated with filtration processes can have a marked effect on the evolution of perturbations in ducts with porous and permeable walls is, as a rule, beyond the applicability range of the model considered.

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